Supplementary File 1 Supplementary methods to this study.

In the CES function, I express the output (y) as a function of the fraction of labor or investment (v) allocated to a sector producing an input:

$$y=\left[v^{\frac{\sigma-1}{\sigma}}+c^{\frac{\sigma-1}{\sigma}}\cdot(1-v)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{S1}$$

The principal of output maximization delivers a first-order differentiation function:

$$\frac{\partial y}{\partial v}=y^{\frac{1}{\sigma}}\cdot\left[v^{-\frac{1}{\sigma}}-c^{\frac{\sigma-1}{\sigma}}\cdot(1-v)^{-\frac{1}{\sigma}}\right]=0 \tag{S2}$$

The optimal allocation of labor or investment (v_p) leads to the maximal output (y_{max}):

$$v_o=\frac{1}{1+c^{\sigma-1}} \tag{S3}$$

$$y_{max}=\left[v_o^{\frac{\sigma-1}{\sigma}}+c^{\frac{\sigma-1}{\sigma}}\cdot(1-v_o)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}=\left(v_o^{-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}=v_o^{\frac{1}{1-\sigma}} \tag{S4}$$

In Equations 1–6, the initial output (Y_0) produced by energy (E_0) and non-energy product (B_0) is a function of the initial capital (K_0), the initial labor (L_0), the initial fraction of labor ($j_{e,0}$) and investment ($i_{e,0}$) allocated to the sectors producing energy products, the initial fraction of labor ($j_{g,0}$) and investment ($i_{g,0}$) in the energy sector that are allocated to produce renewable energy, the initial energy-use efficiency ($\eta_{e,0}$), the initial efficiency of producing non-energy product ($\beta_{b,0}$), and the initial efficiency of producing fossil fuel ($\beta_{d,0}$) and renewable energy ($\beta_{g,0}$):

$$Y_0^{\frac{\sigma_Y-1}{\sigma_Y}}=B_0^{\frac{\sigma_Y-1}{\sigma_Y}}+\left(\eta_{e,0}\cdot E_0\right)^{\frac{\sigma_Y-1}{\sigma_Y}}=\left\{\beta_{b,0}\cdot\left[K_0\cdot(1-i_{e,0})\right]^{\gamma}\cdot\left[L_0\cdot(1-j_{e,0})\right]^{1-\gamma}\right\}^{\frac{\sigma_Y-1}{\sigma_Y}}+$$

$$\left(\eta_{e,0}\cdot K_0^{\gamma}\cdot i_{e,0}^{\gamma}\cdot L_0^{1-\gamma}\cdot j_{e,0}^{1-\gamma}\right)^{\frac{\sigma_Y-1}{\sigma_Y}}\cdot\left\{\left[\beta_{d,0}\cdot(1-i_{g,0})^{\gamma}\cdot(1-j_{g,0})^{1-\gamma}\right]^{\frac{\sigma_E-1}{\sigma_E}}+\left(\beta_{g,0}\cdot i_{g,0}^{\gamma}\cdot j_{g,0}^{1-\gamma}\right)^{\frac{\sigma_E-1}{\sigma_E}}\right\}^{\frac{\sigma_E}{\sigma_E-1}\cdot\frac{\sigma_Y-1}{\sigma_Y}} \tag{S5}$$

I re-formulate this equation by moving capital and labor to the left side of the equation:

$$\left(\frac{Y_0}{K_0^{\gamma}\cdot L_0^{1-\gamma}}\right)^{\frac{\sigma_Y-1}{\sigma_Y}}=\left[\beta_{b,0}\cdot(1-i_{e,0})^{\gamma}\cdot(1-j_{e,0})^{1-\gamma}\right]^{\frac{\sigma_Y-1}{\sigma_Y}}+$$

$$\left(\eta_{e,0}\cdot i_{e,0}^{\gamma}\cdot j_{e,0}^{1-\gamma}\right)^{\frac{\sigma_Y-1}{\sigma_Y}}\cdot\left\{\left[\beta_{d,0}\cdot(1-i_{g,0})^{\gamma}\cdot(1-j_{g,0})^{1-\gamma}\right]^{\frac{\sigma_E-1}{\sigma_E}}+\left(\beta_{g,0}\cdot i_{g,0}^{\gamma}\cdot j_{g,0}^{1-\gamma}\right)^{\frac{\sigma_E-1}{\sigma_E}}\right\}^{\frac{\sigma_E}{\sigma_E-1}\cdot\frac{\sigma_Y-1}{\sigma_Y}} \tag{S6}$$

Let $L_{d,0}$, $L_{g,0}$, and $L_{b,0}$ denote the initial labor producing fossil fuel, renewable energy, and non-energy products, respectively. Let $K_{d,0}$, $K_{g,0}$, and $K_{b,0}$ denote the initial capital in the sectors producing fossil fuel, renewable energy, and non-energy products, respectively. A duality form in the CES function leads to the same allocation of labor, investment, and thus the capital in the equilibrium state:

$$i_{e,t}=j_{e,t}=(L_{d,0}+L_{g,0})/(L_{d,0}+L_{g,0}+L_{b,0})=(K_{d,0}+K_{g,0})/(K_{d,0}+K_{g,0}+K_{b,0}) \tag{S7}$$

$$i_{g,t}=j_{g,t}=L_{g,0}/(L_{d,0}+L_{g,0})=K_{g,0}/(K_{d,0}+K_{g,0}) \tag{S8}$$

As a result, optimizing the allocation of labor and investment between fossil fuel and renewable energy under output maximization by applying Equations S3, S4 for Equation S6 delivers a first-order differentiation function:

$$\frac{\partial Y_0}{\partial i_{g,0}}=\frac{\partial Y_0}{\partial j_{g,0}}=0 \Longrightarrow i_{g,0}=j_{g,0}=\frac{1}{1+\left(\frac{\beta_{d,0}}{\beta_{g,0}}\right)^{\sigma_E\text{-}1}} \tag{S9}$$

The ratio of the initial production of renewable energy (G_0) to the initial production of fossil fuel (F_0) is derived as:

$$\frac{G_0}{F_0}=\frac{\beta_{g,0}\cdot i_{g,0}{}^{\gamma}\cdot j_{g,0}{}^{1\text{-}\gamma}}{\beta_{d,0}\cdot(1\text{-}i_{g,0})^{\gamma}\cdot(1\text{-}j_{g,0})^{1\text{-}\gamma}}=\left(\frac{i_{g,0}}{1\text{-}i_{g,0}}\right)^{\frac{\sigma_E}{\sigma_E\text{-}1}}=\left(\frac{\beta_{g,0}}{\beta_{d,0}}\right)^{\sigma_E} \tag{S10}$$

The useful energy product is derived as:

$$E_0=\left(K_0\cdot i_{e,0}\right)^{\gamma}\cdot\left(L_0\cdot j_{e,0}\right)^{1\text{-}\gamma}\cdot\beta_{g,0}\cdot i_{g,0}{}^{\frac{1}{1\text{-}\sigma_E}} \tag{S11}$$

Substituting Equation S9 into Equation S6 returns the output as a function of the fraction of labor ($j_{e,0}$) and investment ($i_{e,0}$) allocated to the energy sector:

$$\left(\frac{Y_0}{K_0{}^{\gamma}\cdot L_0{}^{1\text{-}\gamma}}\right)^{\frac{\sigma_Y\text{-}1}{\sigma_Y}}=\left[\beta_{b,0}\cdot(1\text{-}i_{e,0})^{\gamma}\cdot(1\text{-}j_{e,0})^{1\text{-}\gamma}\right]^{\frac{\sigma_Y\text{-}1}{\sigma_Y}}+\left(\eta_{e,0}\cdot i_{e,0}{}^{\gamma}\cdot j_{e,0}{}^{1\text{-}\gamma}\cdot\beta_{g,0}\cdot i_{g,0}{}^{\frac{1}{1\text{-}\sigma_E}}\right)^{\frac{\sigma_Y\text{-}1}{\sigma_Y}} \tag{S12}$$

I apply Equations S3, S4 for Equation S12 to optimize the fraction of labor ($j_{e,0}$) and investment ($i_{e,0}$) allocated to the energy sector under output maximization:

$$\frac{\partial Y_0}{\partial i_{e,0}}=\frac{\partial Y_0}{\partial j_{e,0}}=0 \Longrightarrow i_{e,0}=j_{e,0}=\frac{1}{1+\left(\frac{\beta_{b,0}}{\eta_{e,0}\cdot\beta_{g,0}\cdot i_{g,0}{}^{\frac{1}{1\text{-}\sigma_E}}}\right)^{\sigma_Y\text{-}1}} \tag{S13}$$

Substituting Equation S13 into Equation S12 returns the initial output:

$$Y_0=\left(\eta_{e,0}\cdot\beta_{g,0}\cdot i_{g,0}{}^{\frac{1}{1\text{-}\sigma_E}}\cdot i_{e,0}{}^{\frac{1}{1\text{-}\sigma_Y}}\right)\cdot K_0{}^{\gamma}\cdot L_0{}^{1\text{-}\gamma} \tag{S14}$$

When the capital growth reaches an equilibrium, the initial capital (K_0) is a function of the initial output (Y_0):

$$\frac{dK_t}{dt}=I_0\text{-}\theta\cdot K_0=i_{s,0}\cdot Y_0\text{-}\theta\cdot K_0=0 \tag{S15}$$

Then, the initial efficiency of producing renewable energy ($\beta_{g,0}$) is derived as:

$$\beta_{g,0}=E_0/\left[\left(K_0\cdot i_{e,0}\right)^{\gamma}\cdot\left(L_0\cdot j_{e,0}\right)^{1\text{-}\gamma}\cdot i_{g,0}{}^{\frac{1}{1\text{-}\sigma_E}}\right] \tag{S16}$$

The initial energy-use efficiency ($\eta_{e,0}$) is derived as:

$$\eta_{e,0}=Y_0/\left[\left(\beta_{g,0}\cdot i_{g,0}{}^{\frac{1}{1\text{-}\sigma_E}}\cdot i_{e,0}{}^{\frac{1}{1\text{-}\sigma_Y}}\right)\cdot K_0{}^{\gamma}\cdot L_0{}^{1\text{-}\gamma}\right] \tag{S17}$$

I calibrate the ratio of the initial efficiency producing fossil fuel to the initial efficiency producing renewable energy ($\beta_{d,0}/\beta_{g,0}$) to achieve a 10% share of renewable energy in the global energy consumption in 2000 (G_0/D_0=0.1). I calibrate the ratio of the initial efficiency producing renewable energy to the initial efficiency producing non-energy products ($\beta_{g,0}/\beta_{b,0}$) to achieve a 50% fraction of total investment allocated to the energy sectors ($i_{e,0}$=0.5).