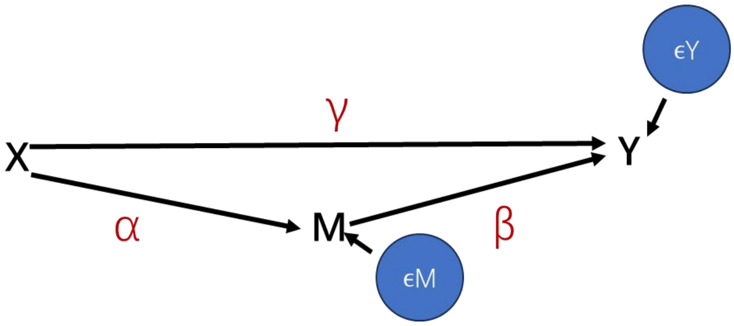
Supplementary File 1 Structural equation models (SEMs)

Structural equation models (SEMs) are a robust technique that combines multiple regression analysis, factor analysis, and path analysis. This integrated approach enables researchers to test complex theoretical models that specify relationships among both observed (measured) and latent (unobserved) variables [1].

The structural equation model employs maximum likelihood estimation to optimize the model parameters, aiming to maximize the probability (i.e., likelihood) of the observed data given the parameters and achieve optimal alignment between the model and actual observations [2].



In the above schematic diagram of SEMs, variable X is considered the independent variable, M serves as an intermediate variable, and Y represents the dependent variable. The path coefficients α, β, and γ are used to quantify the relationships between these variables. Additionally, ϵM and ϵY denote the variances of M and Y, correspondingly. To accurately estimate the path coefficient, the maximum likelihood estimation method was employed in Eqs. (S1) and (S2):

$$M = \alpha X + \epsilon M \tag{S1}$$

$$Y = \beta X + \gamma M + \epsilon Y \tag{S2}$$

Where, Eqs. (S1) and (S2) represent the model equations, where it is assumed that the error terms ϵM and ϵY are normally distributed with mean 0 and standard deviations σM and σY, respectively. For an individual observation i, one component of the likelihood function can be expressed by Eq. (S3):

$$L(\alpha, \sigma_m | X_i, M_i) = \frac{1}{\sqrt{2\pi\sigma_M^2}} exp\left(-\frac{(M_i - \alpha X_i)^2}{2\sigma_M^2}\right) \tag{S3}$$

The logarithmic likelihood function in Eq. (S4) is obtained by taking the product of all observations followed by applying the natural logarithmic：

$$LogL(\alpha, \sigma_M | X, M) = -\frac{n}{2} log\left(2\pi\sigma_M^2\right) - \frac{1}{2\sigma_M^2} \sum_{i=1}^{n} (M_i - \alpha X_i)^2 \tag{S4}$$

For models Y and M, the formula Eq. (S5) of the log-likelihood function is:

$$LogL\left(\beta, \gamma, \sigma_y \middle| M, X, Y\right) = -\frac{n}{2} log\left(2\pi\sigma_Y^2\right) - \frac{1}{2\sigma_Y^2} \sum_{i=1}^{n} (Y_i - \beta M_i - \gamma X_i)^2 \tag{S5}$$

Based on the above Eqs. (S4) and (S5), the gradient descent method was employed to optimize the values of parameters α, β, γ, σM, and σY in order to maximize the log-likelihood function of the whole model. Subsequently, each parameter of log L was perturbed and its derivative was equated to zero, leading to the estimation of parameter values by solving these equations.

Significance evaluation was conducted using the information matrix derived from the established structural equation model. The Eqs. (S6-1), (S6-2) and (S6-3) are as follows:

$$t_\alpha = \frac{\alpha}{SE(\alpha)=\sqrt{\frac{\Sigma(M-\widehat{M})^2/(n-2)}{\Sigma(X_i-X)^2}}} \qquad \text{(S6-1)}$$

$$t_\beta = \frac{\beta}{SE(\beta)=\sqrt{\frac{\Sigma(Y-\widehat{Y})^2/(n-2)}{\Sigma(M-\overline{M})^2}}} \qquad \text{(S6-2)}$$

$$t_\gamma = \frac{\beta}{SE(\gamma)=\sqrt{\frac{\Sigma(Y-\widehat{Y})^2/(n-2)}{\Sigma(X-\overline{X})^2}}} \qquad \text{(S6-3)}$$

Where n is the sample size; $\widehat{Y}$ and $\widehat{M}$ are the values predicted by the model; and $\overline{X}$ and $\overline{M}$ are the averages of X and M. The total standardized effects (sum of the direct and indirect effects of standardization) and path coefficients were estimated by using the maximum likelihood algorithm [3].

In brief, the arrow in SEMs represents causality, while the numerical value on the arrow indicates the total standardized effects that quantify the magnitude of influence. Non-significant chi-square test ($p > 0.05$), chi-square to the degree of freedom (χ^2/df < 3), and low root mean square errors of approximation (RMSEA < 0.08) were calculated as the overall goodness of fit for SEMs [4,5].

References

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